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ROLE OF GEOMETRY IN MODERN PHYSICS

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The role of extra dimensions is discussed. It is demonstrated that the idea of extra space in combination with the higher derivative gravity provides appropriate tool for a solution of fundamental problems of the modern physics.

Keywords: Extra space, dimensionality, gravity, fine tuning.

1. Introduction

What is unsatisfactory in the physics?
Too many parameters taken from nowhere. 40-50 - too much to be attractive!
Number of generations, symmetries. Why three generation? How the Nature managed to insert symmetries in Lagrangian?
Fine-tuning of the parameters and their smallness.

Short list of well known small parameters is: electron to proton mass ratio, the inflaton mass to the Planck mass ratio, neutrino to electron mass ratio and others. There are many attempts to solve each problem separately. Warped geometry is used for the solution of small cosmological constant problem. The hybrid inflation has been developed to avoid the smallness of the inflaton mass. The electron to proton mass ratio is discussed in. Seesaw mechanism is usually applied to explain the smallness of neutrino to electron mass ratio. The simplest way to insert the cosmological constant (CC) is to assume inequality $V_{min} > 0$ for a scalar field potential. In this case it is not clear why V_{min} remains so small after quantum corrections are taken into account.

The fine-tuning concerns the narrowness of intervals for the observed parameters. Small deviations from their values are crucial for our Universe. It is implicitly assumed that the latter are formed «somehow in the Beginning» and it is unclear why the parameter values are settled in such narrow intervals to form our Universe. The anthropic principle is often used for the solution of the fine-tuning problem. This nice idea has only one shortcoming - nothing is usually said about the key issue - the origin of a huge number of universes containing the observable one. The landscape idea provides a huge variety of universes. The string theory, the basis of this idea, leads to low energy parameters which are different in diverse universes even if primary parameters are fixed. Unfortunately this set of low energy parameters is a countable one so there is no assurance in success because the parameter values of different universes can be distributed non-uniformly.

The ultimate example of the fine-tuning is the smallness of the cosmological constant (CC). Observations indicate that the current acceleration is described by the general relativity with the extremely small CC. Many attempts have been made to clarify this issue, see e.g. [1, 2, 3].

Increasing role of geometry

Newton - Gravity and Physics are not connected.

Einstein - Gravity gives new objects (BH) and ruled expansion of the Universe.

1980th - Gravity is responsible for the formation of Universe. Extensions of the gravity theory.

2000th - extra dimensions extensively investigated.

3.12.2017 - Nowadays, it is widely accepted that the physics begins at high energies where gravity dominates.

2. Abilities of extra dimensions and the gravity with higher derivatives

Why don't we see the extra dimensions?

The Brane idea raises questions - how was the brane formed? Why matter is trapped in the brane?

Compact Extra Space (ES) with small size $< 10^{18}$ cm. What keeps the size being small?

2.1. Compact extra dimensions

Let us consider a direct product $M_4 \times V_2$ of a 4-dim space M_4 and 2-dim compact space V_2

$$ds^2 = g_{6,AB} dz^A dz^B = g_{4,\mu\nu}(x) dx^\mu dx^\mu + \phi^2(x) g_{2,ab}(y) dy^a dy^b.$$

Here $g_{4,\mu\nu}(x)$ and $g_{2,ab}(y)$ are metrics of the manifolds M_4 and V_2 respectively. x and y are the coordinates of the subspaces M_4 and V_2. We will refer to 4-dim space M_4 and 2-dim compact space V_2 as the main space and an extra space respectively. The metric has the signature (+ - - - ...), the Greek indices $\mu, \nu = 0,1,2,3$ refer to 4-dimensional coordinates. Latin indices run over $a, b = 4,5$.

The action is chosen in the form

$$S = \frac{m_D^4}{2} \int d^6 Z \sqrt{|G|} \left[F(R) + c_1 R_{AB} R^{AB} \right], \tag{1}$$

$$F(R) = R + cR^2 - 2\Lambda. \tag{2}$$

After some manipulations with action (1) we obtain the effective action in the Einstein frame

$$S_{eff} = \frac{v_2}{2} \int d^4 x (\text{sign} F') \left[R_{(4)} + \frac{k(\phi)}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \tag{3}$$

$$k(\phi) = \frac{1}{\phi} \left[3\phi^2 \left(\frac{F''}{F} \right)^2 - 2\phi \left(\frac{F''}{F} \right) + 2 \right], \tag{4}$$

$$V(\phi) = -sign(1 + 2c\phi) \frac{1}{2} \frac{|\phi| [(c + c_1/2)\phi^2 + \phi - 2\Lambda]}{(1 + 2c\phi)^2}. \tag{5}$$

Here $m_D = 1$ and the Planck mass $M_{Pl} = \sqrt{v_2}$. v_2 is the volume of 2-dim sphere of unit radius.

The potential density $V(\phi)$ represented in Fig.1 depends on the scalar field which is connected to the Ricci scalar $R_{(2)}$ of the extra space, $\phi(x) \equiv R_{(2)} = 2e^{-2\beta(x)}$. The presence of the potential minimum indicates stationarity of extra space of constant curvature.

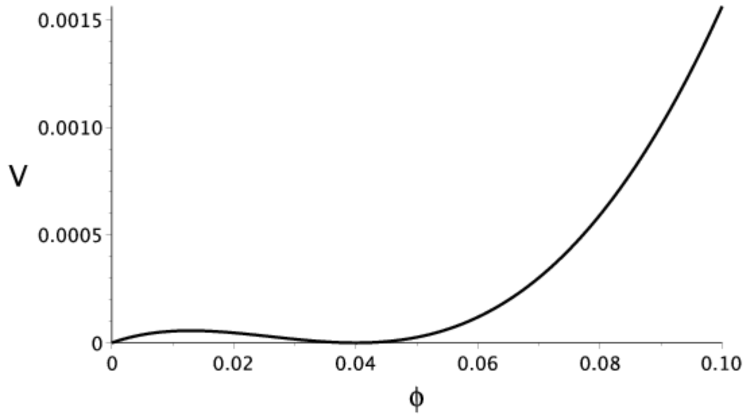

Fig. 1. The plot of the potential density V. The Lagrangian parameters $c = 5, \Lambda = 0.01, c_1 = -27$.

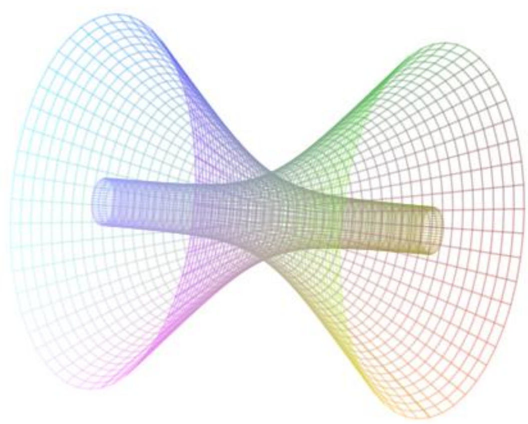

Fig. 2. Interpenetrating spaces in the spherical coordinates look like two intersecting funnels.

2.2. Funnel solutions — a bridge to large 4-dim spaces

Let us specify a geometry of the space and consider the space $\mathbb{V}_D$ with $D = 6$ and metric in the form

$$ds^2 = e^{2\alpha(u)}dt^2 - du^2 - e^{2\beta_1(u)}G_{1,ab}dy^a dy^b - e^{2\beta_2(u)}G_{2,mn}dz^m dz^n \qquad (6)$$

where $-\infty < u < \infty$. There are three independent functions: $\beta_1(u)$, $\beta_2(u)$ and the redshift function $\alpha(u)$. The variable u is a proper distance coordinate. The 2-dim subspaces $\mathbb{W}_{1,2}$ are described by coordinates y_a, z_m $(a = 3,4; m = 5,6)$ and represent two spheres of radius $r_1(u) = e^{\beta_1(u)}$ and $r_2(u) = e^{\beta_2(u)}$. One of the solution represented in Fig. 2 is stable one. There are no horizons so that signals could be sent from one universe to another.

2.3. The role of the postulates in the explaining of our Universe properties

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There are three aspects that must be clarified by a future theory. They are

- Metric of extra space,
- Fine-tuning of the physical parameters,
- Formation of symmetries.

The most economical way that could lead to success starts by admitting the following postulates:

- Extra space does exist. Number of extra dimensions is yet arbitrary.
- The gravitational Lagrangian contains higher derivatives. $f(R)$ gravity helps to stabilize the size of ES.
- Quantum fluctuations of space-time are important at high energies. Space-time foam produces all kinds of manifolds

General picture of the Universe formations looks as follows.

Space-time foam (quantum fluctuations of metric) produces continuum set of manifolds. Small part of them evolves into universes with compact extra space and large 4-dim space. Some of them are appropriate for the intelligent life.

It is known that for complicated structures (life) to be formed, all physical parameters should be **fine tuned**. It means that a measure of universes endowed by intelligent life is very small.

In the framework of this approach, all physical parameters depend on a metric of the extra space. It is assumed that the metric is quite complicated and its study is the challenging problem for future.

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РОЛЬ ГЕОМЕТРИИ В СОВРЕМЕННОЙ ФИЗИКЕ

С.Г Рубин

В работе обсуждается роль дополнительных измерений. Демонстрируется, что идея дополнительного пространства в комбинации с теорией гравитации с высшими производными является подходящим инструментом для решения фундаментальных проблем современной физики.

Ключевые слова: Дополнительное пространство, размерность, гравитация, тонкая настройка.

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ON CONFORMAL KILLING AND HARMONIC FORMS ON RIEMANNIAN SYMMETRIC SPACES

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We prove that there are no non-zero closed and coclosed conformal Killing L^2-forms on a complete, simply connected and irreducible symmetric space with negative scalar curvature. We also prove that there are no non-zero harmonic forms on a complete, simply connected and irreducible symmetric space with positive scalar curvature.

Keywords: Riemannian symmetric space, harmonic forms, closed and coclosed conformal Killing forms.

Conformal Killing forms have been defined on Riemannian manifolds more than forty-five years ago by Tachibana (see [1]) as a natural generalization of conformal Killing vector fields. Surveys of the publications on these forms can be found in the introduction to our last paper [2].

A Riemannian globally symmetric space of *non-compact type* (M,g) is complete and also (M,g) has a nonpositive sectional curvature. We also know that a Riemannian symmetric space has nonpositive (resp. non-negative) curvature operator if and only if it has nonpositive (resp. non-negative) sectional curvature (see [3]). Note that symmetric spaces of non-compact type are non-compact. After the above remarks, the assertion of the following theorem becomes obvious.

Theorem 1. *Let (M,g) be an n-dimensional $(n \geq 3)$ complete, simply connected symmetric space. If (M,g) is irreducible and its scalar curvature is negative, then there are no non-zero closed and coclosed conformal Killing L^2-forms of degree p $(1 \leq p \leq n-1)$ on (M,g).*

Theorem 2. *Let (M,g) be an 2p-dimensional $(p \geq 1)$ complete, simply connected symmetric space. If (M,g) is irreducible and its scalar curvature is negative, then there are no non-zero conformal Killing L^2-forms of degree p on (M,g).*

It is well known that a Riemannian globally symmetric space of *compact type* (M,g) is compact and also (M,g) has a nonpositive sectional curvature (see [3]). Then the following theorem holds.